

CHAPTER - 8

PLANE WAVE PROPAGATION

" CHAPTER - 8 PLANE-WAVE PROPAGATION "

Exercise 8.1

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{and} \quad \nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E}) \Rightarrow$$

$$\nabla (\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{or} \quad \nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

Exercise 8.2

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu \sigma \frac{\partial E_x}{\partial t} + \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu \sigma \frac{\partial E_y}{\partial t} + \mu \epsilon \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu \sigma \frac{\partial E_z}{\partial t} + \mu \epsilon \frac{\partial^2 E_z}{\partial t^2}$$

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = \mu \sigma \frac{\partial H_x}{\partial t} + \mu \epsilon \frac{\partial^2 H_x}{\partial t^2}$$

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_y}{\partial z^2} = \mu \sigma \frac{\partial H_y}{\partial t} + \mu \epsilon \frac{\partial^2 H_y}{\partial t^2}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = \mu \sigma \frac{\partial H_z}{\partial t} + \mu \epsilon \frac{\partial^2 H_z}{\partial t^2}$$

Exercise 8.3

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow -\mu \frac{\partial \vec{H}}{\partial t} = -\frac{\partial E_y}{\partial z} \vec{a}_x + \frac{\partial E_x}{\partial z} \vec{a}_y$$

For $E_x = E_x(t - z/u)$ and $E_y = E_y(t - z/u)$, $d/dt \rightarrow 1$ and $d/dz \rightarrow -1/u$

Then

$$-\mu H_x = \frac{1}{u} E_y \Rightarrow H_x = -\frac{\sqrt{\mu \epsilon}}{\mu} E_y = -\sqrt{\frac{\epsilon}{\mu}} E_y = -\frac{E_y}{\eta} \quad u = \frac{1}{\sqrt{\mu \epsilon}}$$

$$-\mu H_y = -\frac{1}{u} E_x \Rightarrow H_y = \sqrt{\frac{\epsilon}{\mu}} E_x = \frac{E_x}{\eta}$$

$$\text{Thus, } \eta [H_x \vec{a}_x + H_y \vec{a}_y] = -E_y \vec{a}_x + E_x \vec{a}_y$$

$$\text{or } \eta \vec{H} = \vec{a}_z \times \vec{E}$$

Exercise 8.4 $\vec{E} = 100 \sin(10^8 t + x/\sqrt{3}) \vec{a}_z$ V/m

Verify $\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ ① $\nabla^2 \vec{E} = \frac{\partial^2 E_z}{\partial x^2} = -\frac{100}{3} \sin(10^8 t + x/\sqrt{3})$

and $\frac{\partial^2 E_z}{\partial t^2} = -100 \times 10^{16} \sin(10^8 t + x/\sqrt{3})$

From ① $\mu \epsilon = \frac{10^{16}}{3} \Rightarrow \epsilon_r = \frac{10^{16}}{3} \times (3 \times 10^8)^2 = 3$ Hence $\epsilon_r = 3$

(b) $10^8 t + x/\sqrt{3} = \text{Const} \Rightarrow \frac{dx}{dt} = -\sqrt{3} \times 10^8$ Hence $\vec{u} = -\sqrt{3} \times 10^8 \vec{a}_x$ m/s

$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow \mu \frac{\partial \vec{H}}{\partial t} = \frac{\partial E_z}{\partial x} \vec{a}_y$ or $H_y = 0.4594 \sin(10^8 t + x/\sqrt{3})$ A/m

$\vec{S} = \vec{E} \times \vec{H} = -45.94 \sin^2(10^8 t + x/\sqrt{3}) \vec{a}_x$ W/m²

$\langle \vec{S} \rangle = \frac{1}{T} \int_0^T \vec{S} dt = -22.97 \vec{a}_x$ W/m²

Exercise 8.5 $\vec{E} \perp \vec{H}$ & $\vec{E} \cdot \vec{H} = 0$ $\vec{H} = \frac{1}{\eta} (\vec{a}_z \times \vec{E})$

Using Vector identity $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A})$

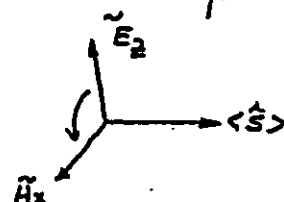
$\vec{E} \cdot \vec{H} = \frac{1}{\eta} [\vec{E} \cdot (\vec{a}_z \times \vec{E})] = \frac{1}{\eta} [\vec{a}_z \cdot (\vec{E} \times \vec{E})] = 0$

Exercise 8.6 $\omega = 2\pi \times 10^9$ rad/s $\beta_0 = \frac{\omega}{c} = 20.94$ rad/m $\eta_0 \approx 377 \Omega$

$\vec{E} = 120 e^{-j20.94y} \vec{a}_z$ V/m

For y-directed propagation,

$\vec{H} = 0.318 e^{-j20.94y} \vec{a}_x$ A/m



$\langle \vec{S} \rangle = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*] = 19.1 \vec{a}_y$ W/m²

Exercise 8.7 $\omega = 200\pi \times 10^6$ rad/s $\beta_0 = \frac{\omega}{c} = \frac{2}{3}\pi$ rad/m, $\eta_0 \approx 377 \Omega$

$\vec{u}_p = -3 \times 10^8 \vec{a}_z$ m/s

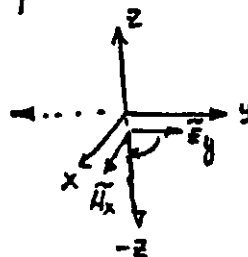
$\lambda_0 = \frac{2\pi}{\beta_0} = 3$ m

$\vec{H} = 0.1 e^{j\beta_0 z} \vec{a}_x$ A/m

$\vec{E} = 37.7 e^{j\beta_0 z} \vec{a}_y$ V/m

$\vec{J}_d = j\omega \epsilon_0 \vec{E} = j0.209 e^{j\beta_0 z} \vec{a}_y$ A/m²

$\langle \vec{S} \rangle = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*] = -1.885 \vec{a}_z$ W/m²



Exercise 8.8 $\nabla \times \vec{E} = -j\omega\mu\vec{H}$, $\nabla \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E} = j\omega\hat{\epsilon}\vec{E}$ $\hat{\epsilon} = \epsilon[1 - j\frac{\sigma}{\omega\epsilon}]$
 $\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = 0$ $\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0$

$\nabla \times \nabla \times \vec{E} = -j\omega\mu(\nabla \times \vec{H}) \Rightarrow \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\omega^2\mu\hat{\epsilon}\vec{E}$

$\nabla^2 \vec{E} = \hat{\gamma}^2 \vec{E}$ where $\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}}$. Similarly, $\nabla^2 \vec{H} = \hat{\gamma}^2 \vec{H}$

Exercise 8.9 $\hat{\epsilon} = \epsilon[1 - j\tan\phi] = \epsilon \sec\phi \angle -\phi$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = j\omega\sqrt{\mu\epsilon \sec\phi} \angle -\phi/2 = \omega\sqrt{\mu\epsilon \sec\phi} [\sin\phi/2 + j\cos\phi/2]$

Thus $\alpha = \omega\sqrt{\mu\epsilon \sec\phi} \sin\phi/2$ and $\beta = \omega\sqrt{\mu\epsilon \sec\phi} \cos\phi/2$

Exercise 8.10 $f = 100 \text{ MHz}$, $\omega = 6.28 \times 10^8 \text{ rad/s}$, $\sigma = 9.375 \text{ mS/m}$, $\epsilon_r = 2.25$
 $\tan\phi = \frac{\sigma}{\omega\epsilon} = 0.75 \Rightarrow \hat{\epsilon} = \epsilon[1 - j0.75]$ $\mu_r = 1$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = j\omega\sqrt{\mu_0\epsilon_0} \sqrt{\mu_r\epsilon_r} \sqrt{1 - j0.75} = 1.11 + j3.33 \Rightarrow \alpha = 1.11 \text{ Np/m}$

$\beta = 3.33 \text{ rad/m}$

$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} \frac{1}{\sqrt{1 - j0.75}} = 224.79 \angle 18.43^\circ \Omega$

$\delta = \frac{1}{\alpha} = 900 \text{ mm}$

$\vec{E}_y = 125 e^{-1.11z} e^{-j3.33z} \text{ V/m}$

$u_p = \frac{\omega}{\beta} = 1.89 \times 10^8 \text{ m/s}$

$\vec{H}_x = -\frac{125}{224.79} e^{-1.11z} e^{-j3.33z} e^{-j18.43^\circ} \text{ A/m}$

$\langle \vec{S} \rangle = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = \frac{1}{2} \times 125 \times \frac{125}{224.79} \cos 18.43^\circ e^{-2.22z} \hat{a}_z$
 $= 32.97 e^{-2.22z} \hat{a}_z \text{ W/m}^2$

Exercise 8.11 $f = 2.4 \times 10^9 \text{ Hz}$, $\omega = 1.508 \times 10^{10} \text{ rad/s}$

$\sigma = 6.1 \times 10^7 \text{ S/m}$, $\epsilon_r = 1$, $\mu_r = 1$ $\tan\phi = \frac{\sigma}{\omega\epsilon} = 4.575 \times 10^8 \Rightarrow \text{Very good conductor}$

$\hat{\epsilon} = -j\frac{\sigma}{\omega} = -j0.004$, $\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = (7.602 + j7.602) \times 10^5$

$\alpha = 7.602 \times 10^5 \text{ Np/m}$

$\beta = 7.602 \times 10^5 \text{ rad/m}$

$R_s = \frac{1}{\sigma\delta} = 0.012 \text{ or } 12 \text{ m}\Omega/\text{m}^2$

$\delta = \frac{1}{\alpha} = 1.315 \mu\text{m} \Rightarrow \text{Thickness} = 5\delta \approx 1.32 \mu\text{m}$

Exercise 8.12 @ 60 MHz: $\omega = 2\pi f = 120\pi \text{ rad/s}$

$\sigma = 3.5 \times 10^7 \text{ S/m}$ $\mu_r = 1$ $\epsilon_r = 1$ $\tan\phi = \frac{\sigma}{\omega\epsilon} = 1.05 \times 10^{16}$ Very good Conductor

γ_{loss} , $\hat{\epsilon} = -j\frac{\sigma}{\omega} = -j9.284 \times 10^4$ $\hat{\gamma} = j\omega\sqrt{\mu}\hat{\epsilon} = 91.052 + j91.052$

$\delta = \frac{1}{\alpha} = 10.983 \text{ mm}$. Since Thickness of $5 \text{ mm} < \delta$, $R_s = \frac{100}{2\pi \lambda 2.54 \times 10^{-2} \times 5 \times 10^{-3}} = 3.58 \text{ m}\Omega$

@ 60 MHz: $\tan\phi = \frac{\sigma}{\omega\epsilon} = 1.05 \times 10^{10}$ (Very good Conductor)

$\hat{\epsilon} = -j\frac{\sigma}{\omega} = -j0.093$

$\hat{\gamma} = j\omega\sqrt{\mu}\hat{\epsilon} = (9.105 + j9.105)10^4$ $\delta = \frac{1}{\alpha} = 0.011 \text{ mm}$ and $R_s = 1.63 \Omega$

Exercise 8.13 $\sqrt{\hat{\epsilon}} = \sqrt{\epsilon} [1 - j\frac{\sigma}{2\omega\epsilon}]^{\frac{1}{2}} = \sqrt{\epsilon} [1 - j\frac{\sigma}{2\omega\epsilon}]$ for $\frac{\sigma}{\omega\epsilon} \ll 1$

$\hat{\gamma} = j\omega\sqrt{\mu}\hat{\epsilon} = j\omega\sqrt{\mu}\epsilon [1 - j\frac{\sigma}{2\omega\epsilon}] = j\omega\sqrt{\mu}\epsilon + \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$

$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} [1 - j\frac{\sigma}{2\omega\epsilon}]^{-1} = \sqrt{\frac{\mu}{\epsilon}} [1 + j\frac{\sigma}{2\omega\epsilon}] = \sqrt{\frac{\mu}{\epsilon}} + j\frac{\sigma}{2\omega\epsilon}\sqrt{\frac{\mu}{\epsilon}} \approx \sqrt{\frac{\mu}{\epsilon}}$

Exercise 8.14 $f = 60 \text{ MHz}$ $\omega = 120\pi \times 10^6 \text{ rad/s}$ $\sigma = 5 \times 10^3 \text{ S/m}$

$\tan\phi = \frac{\sigma}{\omega\epsilon} = 0.094$ (Poor Conductor)

$\mu \rightarrow \mu_0$, $\epsilon \Rightarrow \epsilon_0 (1/6)$

$\hat{\epsilon} = \epsilon [1 - j\frac{\sigma}{\omega\epsilon}] = \epsilon [1 - j0.094]$

$\beta = \omega\sqrt{\mu\epsilon} = 5.027 \text{ rad/m}$

$\hat{\gamma} = j\omega\sqrt{\mu}\hat{\epsilon} = j\omega\sqrt{\mu\epsilon} [1 - j0.047]$

$\alpha = 0.047\beta = 0.236 \text{ Np/m}$

if $\tilde{E}_x = E_0 e^{-\alpha z} e^{-j\beta z}$

$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} \approx \sqrt{\frac{\mu}{\epsilon}} = 94.25 \Omega$

at $z=0$ $\tilde{E}_x(0) = E_0$

at $z=d$ $\tilde{E}_x(d) = E_0 e^{-\alpha d} e^{-j\beta d}$ } $\Rightarrow E_0 e^{-\alpha d} = 0.1 E_0$ $\alpha e^{-\alpha d} = 10$

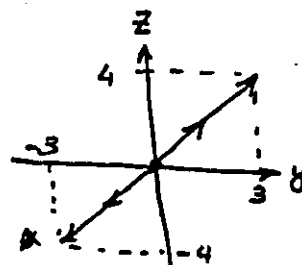
$d = \frac{\ln 10}{\alpha} = 9.76 \text{ m}$

Exercise 8.15 at $\beta x = 45^\circ$

$E_z = 4 \sin \omega t$, $E_y = 3 \sin \omega t$

$\frac{E_y}{E_z} = 0.75 \Rightarrow E_y = 0.75 E_z$

st. line relationship \Rightarrow Linear Polarization



Exercise 8.16 $E_x = 25 e^{j0.4y} \sin(\omega t - 120y)$

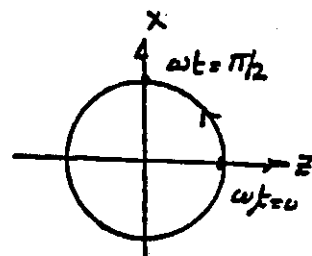
at $y=0$, $E_x = 25 \sin \omega t$ $E_z = 25 \cos \omega t$

$E_x^2 + E_z^2 = 25^2$ [Circular]

$\omega t = 0$ $E_x = 0$, $E_z = 25$

$\omega t = \pi/2$ $E_x = 25$ $E_z = 0$ } Right handed

$E_z = 25 e^{j0.4y} \cos(\omega t - 120y)$



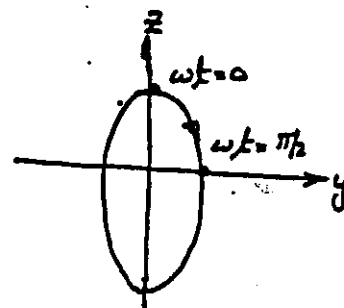
Exercise 8.17 Let $\beta x = 60^\circ$, then

$E_y = 30 \sin \omega t$ $E_z = 40 \cos \omega t$

$(\frac{E_y}{30})^2 + (\frac{E_z}{40})^2 = 1$ [Elliptical]

$\omega t = 0$ $E_y = 0$ $E_z = 40$

$\omega t = \pi/2$ $E_y = 30$ $E_z = 0$ } Left-handed



Exercise 8.18

$\vec{E}_i = 10 e^{-0.92z} e^{-j4.29z} \vec{a}_x$

$\vec{H}_i = \frac{1}{\eta} e^{-0.92z} e^{-j4.29z} e^{-j12.11^\circ} \vec{a}_y$

$\vec{E}_r = \vec{E}_i + \vec{E}_r = [10 e^{-0.92z} e^{-j4.29z} + 4.07 e^{-0.92z} e^{j4.29z} e^{j153.74^\circ}] \vec{a}_x$

$\vec{H}_r = \vec{H}_i + \vec{H}_r = [\frac{1}{\eta} e^{-0.92z} e^{-j4.29z} e^{-j12.11^\circ} - \frac{4.07}{\eta} e^{-0.92z} e^{j4.29z} e^{j141.63^\circ}] \vec{a}_y$

$\vec{E}_t = \vec{E}_2 = 6.6 e^{-5.3z} e^{-j7.45z} e^{j15.81^\circ} \vec{a}_x$

$\vec{H}_t = \vec{H}_2 = 0.153 e^{-5.3z} e^{-j7.45z} e^{-j19.62^\circ} \vec{a}_y$

$\langle \hat{S}_i \rangle = \frac{1}{2} \text{Re}[\vec{E}_i \times \vec{H}_i^*] = 0.543 e^{-1.84z} \vec{a}_z$

$\langle \hat{S}_r \rangle = \frac{1}{2} \text{Re}[\vec{E}_r \times \vec{H}_r^*] = -0.09 e^{-1.84z} \vec{a}_z$

$\langle \hat{S}_2 \rangle = \langle \hat{S}_t \rangle = 0.41 e^{-10.6z} \vec{a}_z$

$\langle \hat{S}_1 \rangle = \frac{1}{2} \text{Re}[\vec{E}_1 \times \vec{H}_1^*] = [0.543 e^{-1.84z} - 0.09 e^{-1.84z} + 0.177 \cos(0.582z) - 0.219 \cos(0.582z)] \vec{a}_z$

at $z=0$ $\langle \hat{S}_1 \rangle = 0.411 \vec{a}_z = \langle \hat{S}_0 \rangle$

at $z=0$

$\langle \hat{S}_i \rangle = 0.543 \vec{a}_z$

$\langle \hat{S}_r \rangle = -0.09 \vec{a}_z$

$\langle \hat{S}_{lr} \rangle = -0.042 \vec{a}_z$

$\langle \hat{S}_{lr} \rangle$

Exercise 8.19 $f = 500 \text{ MHz}$ $\omega = 3.14 \times 10^9 \text{ rad/s}$

$$\epsilon_{r1} = 16 \quad \sigma_1 = 0.02 \Rightarrow \hat{\epsilon}_1 = 1.41 \times 10^{-10} - j 6.37 \times 10^{-12}$$

$$\hat{\gamma}_1 = j\omega\sqrt{\mu_0\hat{\epsilon}_1} = 41.91 \angle 89.71^\circ = 0.94 + j 41.9 \text{ m}^{-1} \quad \alpha_1 = 0.94 \text{ Np/m}, \beta_1 = 41.9 \text{ rad/m}$$

$$\hat{\eta}_1 = \sqrt{\frac{\mu_1}{\hat{\epsilon}_1}} = 94.18 + j 2.12 = 94.2 \angle 1.29^\circ \Omega$$

$$\epsilon_{r2} = 25 \quad \sigma_2 = 0.2 \Rightarrow \hat{\epsilon}_2 = 2.21 \times 10^{-10} - j 6.37 \times 10^{-11}$$

$$\hat{\gamma}_2 = j\omega\sqrt{\mu_0\hat{\epsilon}_2} = 53.41 \angle 81.97^\circ = 7.46 + j 52.89 \text{ m}^{-1} \quad \hat{\eta}_2 = \sqrt{\frac{\mu_2}{\hat{\epsilon}_2}} = 73.91 \angle 8.03^\circ \Omega$$

$$\hat{P} = \frac{\hat{\eta}_2 \cdot \hat{\eta}_1}{\hat{\eta}_1 + \hat{\eta}_2} = 0.13 \angle 154.38^\circ \quad \hat{T} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} = 0.88 \angle 3.78^\circ$$

Max. Value of the incidence field = 10 V/m

$$\vec{E}_i = 0.8 e^{-7.46z} e^{-j52.89z} e^{j3.78^\circ} \vec{a}_x \text{ V/m}$$

$$\vec{H}_i = \frac{0.8}{73.91} e^{-7.46z} e^{-j52.89z} e^{j3.78^\circ} e^{-j8.03^\circ} \vec{a}_y \text{ A/m}$$

$$\langle \hat{S}_i \rangle = \frac{1}{2} \text{Re} [\vec{E}_i \times \vec{H}_i^*] = 0.52 e^{-14.92z} \vec{a}_z \text{ W/m}^2$$

Exercise 8.20

$$u_{p1} = \frac{c}{\beta_1} = 1 \times 10^8 \text{ m/s} \quad \lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{3} \text{ m}$$

$$u_{p2} = \frac{c}{\beta_2} = 1.5 \times 10^8 \text{ m/s} \quad \lambda_2 = \frac{2\pi}{\beta_2} = \pi \text{ m}$$

$$\langle \hat{S}_i \rangle + \langle \hat{S}_r \rangle = [39.79 - 1.59] \vec{a}_z \text{ } \mu\text{W/m}^2 = 38.2 \vec{a}_z \text{ } \mu\text{W/m}^2$$

Exercise 8.21 $\text{SWR} = \frac{|E|_{\max}}{|E|_{\min}} \Rightarrow |E|_{\max} \text{ at } z=0 = E_0 [1 + |P|]$

$$E_{\min} \text{ at } z = \frac{\pi}{2\beta_1} = E_0 [1 - |P|]$$

$$\text{SWR} = \frac{1 + |P|}{1 - |P|}$$

Exercise 8.22

$$\mu_1 = \mu_0, \epsilon_1 = 2.25 \epsilon_0, \mu_2 = \mu_0, \epsilon_2 = 9 \epsilon_0$$

$$\vec{E}_i = 0.25 e^{-j1.5x} \vec{a}_z \quad \vec{H}_i = -\frac{0.25}{80\pi} e^{-j1.5x} \vec{a}_y \quad \left\{ \begin{array}{l} \langle \hat{S}_i \rangle = 124.34 \vec{a}_x \text{ } \mu\text{W/m}^2 \end{array} \right.$$

$$\vec{E}_r = -\frac{0.25}{3} e^{j1.5x} \vec{a}_z \quad \vec{H}_r = \frac{0.25}{240\pi} e^{j1.5x} \vec{a}_y \quad \left\{ \begin{array}{l} \langle \hat{S}_r \rangle = -13.82 \vec{a}_x \text{ } \mu\text{W/m}^2 \end{array} \right.$$

$$\langle \hat{S}_i \rangle + \langle \hat{S}_r \rangle = 110.52 \vec{a}_x \text{ } \mu\text{W/m}^2$$

$$\beta_1 = \omega\sqrt{\mu_0\epsilon_0\epsilon_{r1}} = 1.5 \text{ rad/m}$$

$$\beta_2 = \omega\sqrt{\mu_0\epsilon_0\epsilon_{r2}} = 3 \text{ rad/m}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_{r1}}} = 80\pi, \quad \eta_2 = \frac{120\pi}{3} = 40\pi$$

$$P = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = -\frac{1}{3}, \quad T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2}{3}$$

$$\vec{E}_t = \frac{0.5}{3} e^{-j3x} \vec{a}_z \quad \vec{H}_t = -\frac{0.5}{120\pi} e^{-j3x} \vec{a}_y \quad \left\{ \begin{array}{l} \langle \hat{S}_t \rangle = 110.52 \vec{a}_x \text{ } \mu\text{W/m}^2 \end{array} \right.$$

Exercise 8.23 $\omega = 600 \times 10^6 \text{ rad/s}$ $\mu \rightarrow \mu_0$ $\epsilon \rightarrow \epsilon_0 (16)$

$\beta = \omega \sqrt{\mu \epsilon} = 8 \text{ rad/m}$ $\eta = \frac{120\pi}{4} = 30\pi$ $u_p = \omega/\beta = 7.5 \times 10^7 \text{ m/s}$ $\rho = -1$

$\vec{E}_i = 100 e^{-j8z} \vec{a}_x$ $\vec{E}_r = -100 e^{j8z} \vec{a}_x$ $\vec{E} = \vec{E}_i + \vec{E}_r = -j200 \sin 8z \vec{a}_x$

$\vec{H}_i = \frac{100}{30\pi} e^{-j8z} \vec{a}_y$ $\vec{H}_r = \frac{100}{30\pi} e^{j8z} \vec{a}_y$ $\vec{H} = \vec{H}_i + \vec{H}_r = \frac{200}{30\pi} \cos 8z \vec{a}_y$

$\vec{J}_s(0) = -\vec{a}_z \times (\vec{H}_i - \vec{H}_r) \Rightarrow \vec{J}_s(0) = \frac{200}{30\pi} \vec{a}_x$

$\langle \vec{P} \rangle = \frac{1}{2} \text{Re} [\vec{J}_s(0) \times \vec{E}_i^*(0)] = \frac{1}{2} \left[\frac{200}{30\pi} \cdot \frac{100}{30\pi} (4\pi \times 10^{-7}) \right] = 1.414 \vec{a}_z \text{ } \mu\text{W/m}^2$

$d = \frac{\eta\pi}{\beta}$ For $\eta=1$, $d = \frac{\pi}{8} = 39.27 \text{ cm}$

Exercise 8.24

$E_{ix} = 100 \cos(\omega t - 8z)$

$H_{iy} = \frac{100}{30\pi} \cos(\omega t - 8z)$

$E_{rx} = -100 \cos(\omega t + 8z)$

$H_{ry} = \frac{100}{30\pi} \cos(\omega t + 8z)$

$E_x = 200 \sin 8z \sin \omega t$

$E_y = \frac{200}{30\pi} \cos 8z \cos \omega t$

Exercise 8.25 $\omega = 96 \times 10^6 \text{ rad/s}$ $\epsilon_1 = \epsilon_0$ $\mu_1 = \mu_0$ $\epsilon_2 = 81\epsilon_0$ $\mu_2 = 1$ $\sigma_2 = 4 \text{ S/m}$

$\gamma_1 = j\omega\sqrt{\mu_1\epsilon_1} = j0.32 \text{ m}^{-1}$ $\beta_1 = 0.32 \text{ rad/m}$ $\hat{\epsilon}_2 = \epsilon_0 [1 - j \frac{\sigma_2}{\omega\epsilon_2}]$

$\eta_1 \approx 377 \Omega$ $\hat{\eta}_2 = \sqrt{\frac{\mu_0}{\hat{\epsilon}_2}} = 5.491 \angle 44.51^\circ \Omega$

$\hat{\gamma}_2 = j\omega\sqrt{\mu\hat{\epsilon}_2} = 15.4 + j15.667$

$\hat{\rho} = \frac{\hat{\eta}_2 - \eta_1}{\hat{\eta}_2 + \eta_1} = 0.979 \angle 178.83^\circ$

$\vec{E}_t = 2.9 e^{-15.4z} e^{-j15.667z} e^{j43.93^\circ} \vec{a}_x$

$\vec{H}_t = \frac{2.9}{5.491} e^{-15.4z} e^{-j15.667z} e^{j43.93^\circ} e^{-j44.51^\circ} \vec{a}_y$

$\hat{\Gamma} = 1 + \hat{\rho} = 0.029 \angle 43.93^\circ$

$\langle \hat{S}_t \rangle_z = \frac{2.9^2}{2 \times 5.491} e^{-30.8z} \cos(44.51 - 43.93)$

$\langle \hat{S}_t \rangle = 0.77 e^{-30.8z} \vec{a}_z \text{ W/m}^2$

$\vec{E}_i = 100 e^{-j0.32z} \vec{a}_x \text{ V/m}$

$\vec{H}_i = \frac{100}{377} e^{-j0.32z} \vec{a}_y \text{ A/m}$

$\langle \hat{S}_i \rangle = \frac{1}{2} \text{Re} [\vec{E}_i \times \vec{H}_i^*] = 13.26 \vec{a}_z \text{ W/m}^2$

$\vec{E}_r = 97.9 e^{+j0.32z} e^{j178.83^\circ} \vec{a}_x$

$\vec{H}_r = -\frac{97.9}{377} e^{j0.32z} e^{j178.83^\circ} \vec{a}_y$

$\langle \hat{S}_r \rangle = -12.71 \vec{a}_z \text{ W/m}^2$

$\delta_2 = \frac{1}{\alpha_2} \Rightarrow \delta_2 \approx 6.5 \text{ cm}$

Problem 8.1 $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ $\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$ $\nabla \cdot \vec{B} = 0$ $\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H}) \Rightarrow \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Thus: $\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ Likewise $\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$

Problem 8.2 $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ $\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$ $\nabla \cdot \vec{B} = 0$

$$\nabla \cdot \vec{D} = \rho \Rightarrow$$

$$\nabla \cdot \vec{E} = \rho / \epsilon$$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H}) \Rightarrow$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{or}$$

$$\nabla^2 \vec{E} - \mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{1}{\epsilon} \nabla \rho \quad (1)$$

$$\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E}) \Rightarrow \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{or} \quad \nabla^2 \vec{H} - \mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad (2)$$

Problem 8.3 $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ $\nabla \cdot \vec{B} = 0$ $\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0$, $\nabla \times \vec{H} = \sigma \vec{E}$

$$\left. \begin{array}{l} \nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} \nabla \times \vec{H} \Rightarrow \nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} \\ \nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} \Rightarrow \nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} \end{array} \right\} \text{Similar expressions}$$

Problem 8.4 $\vec{E} = E_0 \cos \omega t \cos \beta z \vec{a}_x$

$$\frac{\partial^2 \vec{E}}{\partial z^2} = -\beta^2 E_0 \cos \omega t \cos \beta z \vec{a}_x \quad \frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 E_0 \cos \omega t \cos \beta z \vec{a}_x$$

For \vec{E} -field to exist: $\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \Rightarrow \beta^2 = \omega^2 \mu \epsilon$ or $\beta = \pm \omega \sqrt{\mu \epsilon}$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow$$

$$\mu \frac{\partial \vec{H}}{\partial t} = -\frac{\partial E_x}{\partial z} \vec{a}_y \quad \text{or} \quad \frac{\partial \vec{H}}{\partial t} = \frac{\beta E_0}{\mu} \cos \omega t \sin \beta z \vec{a}_y$$

Integrate w.r.t. t : $\vec{H} = \frac{\beta E_0}{\omega \mu} \sin \omega t \sin \beta z \vec{a}_y$

Thus:

$$\vec{E} = \frac{1}{2} [E_0 \cos(\omega t - \beta z) + \cos(\omega t + \beta z)] \vec{a}_x$$

$$\vec{H} = \frac{1}{2} \frac{\beta}{\omega \mu} E_0 [\cos(\omega t - \beta z) - \cos(\omega t + \beta z)] \vec{a}_y$$

Problem 8.5 $E_x = F_x(t + \sqrt{\mu\epsilon}z)$

$$\nabla \times \vec{E} = \vec{a}_y \frac{\partial E_x}{\partial z} = \vec{a}_y \frac{\partial F_x}{\partial(t + \sqrt{\mu\epsilon}z)} \sqrt{\mu\epsilon}$$

$$\text{Since } \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow \frac{\partial \vec{H}}{\partial t} = -\sqrt{\frac{\epsilon}{\mu}} \frac{\partial F_x}{\partial(t + \sqrt{\mu\epsilon}z)} \vec{a}_y \quad (1)$$

$$\text{However, } \frac{\partial \vec{H}}{\partial t} = \frac{\partial \vec{H}}{\partial(t + \sqrt{\mu\epsilon}z)} \quad (2). \text{ From (1) and (2) } H_y = -\sqrt{\frac{\epsilon}{\mu}} F_x$$

$$\vec{S} = \vec{E} \times \vec{H} = -\sqrt{\frac{\epsilon}{\mu}} F_x^2 \vec{a}_z$$

Problem 8.6 Let there be an x-component of \vec{E} -field: $E_x = F(t - z/u)$

$$\frac{\partial E_x}{\partial z} = -\frac{1}{u} \frac{\partial F}{\partial(t - z/u)} \quad \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 F}{\partial(t - z/u)^2}$$

$$\frac{\partial E_x}{\partial t} = \frac{\partial F}{\partial(t - z/u)} \quad \frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 F}{\partial(t - z/u)^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\eta = \sqrt{\mu_r \epsilon_r}$$

$$\text{For } \frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} \Rightarrow \frac{1}{u^2} = \mu\epsilon \Rightarrow u = \pm \sqrt{\frac{1}{\mu\epsilon}} = \pm \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{1}{\sqrt{\mu_r \epsilon_r}} = \pm \frac{c}{\eta}$$

Problem 8.7 $\vec{H} = 100 e^{j\beta z} \vec{a}_x \text{ A/m}, \quad \eta_0 = 120\pi \approx 377\Omega$

$$\vec{E} = 37,700 e^{j\beta z} \vec{a}_y \text{ V/m} \quad u = -3 \times 10^8 \vec{a}_z \text{ m/s} \quad \omega = 30,000 \text{ rad/s}$$

$$\beta_0 = \frac{\omega}{c} = 0.0001 \text{ (rad/m)} \Rightarrow \lambda_0 = \frac{2\pi}{\beta_0} = 62.83 \text{ km}$$

$$\begin{aligned} \langle \vec{S} \rangle &= -\frac{1}{2} \times 37,700 \times 100 \vec{a}_z \\ &= 1.885 \vec{a}_z \text{ MW/m}^2 \end{aligned}$$

Problem 8.8 $f = 100 \text{ MHz} \quad \omega = 2\pi f = 6.28 \times 10^8 \text{ rad/s} \quad \mu_r = 1 \quad \epsilon_r = 4$

$$\vec{E}_x = 500 e^{-j\beta y} \text{ V/m} \quad u_p = \frac{c}{\eta} = 1.5 \times 10^8 \text{ m/s} \quad \eta = \sqrt{\mu_r \epsilon_r} = 2$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} = 60\pi \Omega$$

$$\beta = \frac{\omega}{u_p} = 4.19 \text{ rad/m}$$

$$\vec{H}_z = -\frac{500}{60\pi} e^{-j\beta y} \text{ A/m}$$

$$\langle \hat{S} \rangle = \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*] = 663.15 \vec{a}_y \text{ W/m}$$

$$\langle P \rangle = \int_S \langle \hat{S} \rangle \cdot d\vec{s} = 1.06 \text{ W}$$

Problem 8.9 Assume $\vec{E} = \vec{E}_x \vec{a}_x$ and $\vec{U}_p = U_p \vec{a}_z$ $\mu_r = 1$ $\epsilon_r = 2.5$

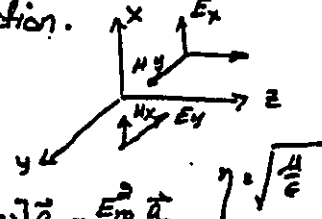
Then $\vec{E} = 100 e^{-j\beta z} \vec{a}_x$ kV/m $\eta = \sqrt{\mu_r \epsilon_r} = 1.581$ $U_p = \frac{E}{\eta} = 1.897 \times 10^8$ m/s

$\vec{H} = 0.42 e^{-j\beta z} \vec{a}_y$ kA/m. $\beta = \frac{\omega}{U_p}$, $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{2.5}} = 238.43 \Omega$

$\langle \vec{S} \rangle = \frac{1}{2} \times 100 \times 0.42 \vec{a}_z = 21 \vec{a}_z$ MW/m²

Problem 8.10 Since $\vec{E} = E_m \cos(\omega t - \beta z) \vec{a}_x - E_m \sin(\omega t - \beta z) \vec{a}_y$,
the wave propagates in the z-direction.

Thus: $\vec{H} = \frac{E_m}{\eta} \cos(\omega t - \beta z) \vec{a}_y$
 $+ \frac{E_m}{\eta} \sin(\omega t - \beta z) \vec{a}_x$



$\vec{S} = \vec{E} \times \vec{H} = \left[\frac{E_m^2}{\eta} \cos^2(\omega t - \beta z) + \frac{E_m^2}{\eta} \sin^2(\omega t - \beta z) \right] \vec{a}_z = \frac{E_m^2}{\eta} \vec{a}_z$ $\eta = \sqrt{\frac{\mu}{\epsilon}}$

Problem 8.11 $\nabla \times \vec{E} = -j\omega\mu\vec{H}$, $\nabla \times \vec{H} = j\omega\epsilon\vec{E}$ $\hat{\epsilon} = \epsilon[1 - j\frac{\sigma}{\omega\epsilon}]$

$\nabla \times \nabla \times \vec{E} = -j\omega\mu(\nabla \times \vec{H})$

$\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = 0$

$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = (-j\omega\mu)(j\omega\hat{\epsilon})\vec{E}$

$\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0$

$\nabla^2 \vec{E} = \hat{\gamma}^2 \vec{E}$ $\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}}$

Similarly, $\nabla^2 \vec{H} = \hat{\gamma}^2 \vec{H}$

Problem 8.12 $\nabla \times \vec{H} = \sigma \vec{E} + j\omega\epsilon\vec{E} = \sigma[1 + j\frac{\omega\epsilon}{\sigma}]\vec{E}$

In a highly conductive medium, $\omega\epsilon \ll \sigma$, we can write above equation as $\nabla \times \vec{H} \cong \sigma \vec{E}$

Thus, $\nabla \times \nabla \times \vec{H} = \sigma(\nabla \times \vec{E})$ Since $\nabla \times \vec{E} = -j\omega\mu\vec{H}$

$\nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -j\omega\mu\sigma \vec{H}$ $\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = 0$

Thus, $\nabla^2 \vec{H} = j\omega\mu\sigma \vec{H} = \hat{\gamma}^2 \vec{H}$ where $\hat{\gamma} = \sqrt{j\omega\mu\sigma}$

$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ Np/m.

$= \sqrt{\omega\mu\sigma} \angle 45^\circ$

$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ rad/m.

$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$ m

$U_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$ m/s

$= \alpha + j\beta$

Problem 8.13 $f = 50 \text{ MHz}$ $\omega = 2\pi f = 3.142 \times 10^8 \text{ rad/s}$ $\epsilon_r = 16$ $\sigma = 0.02 \text{ S/m}$

$$\hat{\epsilon} = \epsilon [1 - j \frac{\sigma}{\omega \epsilon}] = \epsilon [1 - j 0.45]$$

$$\hat{\gamma} = j\omega \sqrt{\mu_0 \epsilon_0} \sqrt{\mu_r \epsilon_r} \sqrt{1 - j 0.45} = j 4.387 \angle -12.12^\circ$$

$$= 4.387 \angle 77.88^\circ = 0.921 + j 4.289$$

$$\hat{\eta} = \sqrt{\frac{\mu_0}{\hat{\epsilon}}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{1}{\epsilon_r}} \sqrt{\frac{1}{1 - j 0.45}}$$

$$= 90 \angle 12.12^\circ \Omega$$

$$\alpha = 0.921 \text{ Np/m} \quad \beta = 4.289 \text{ rad/m}$$

$$\delta = \frac{1}{\alpha} = 1.086 \text{ m}$$

$$u_p = \frac{\omega}{\beta} = 7.325 \times 10^7 \text{ m/s}$$

$$\lambda = \frac{2\pi}{\beta} = 1.465 \text{ m} \quad e^{-\alpha d} = 0.1 \Rightarrow d = 2.5 \text{ m}$$

$$\tilde{E}_x = 120 e^{-\alpha z} e^{-j\beta z} \text{ V/m}, \quad \tilde{H}_y = \frac{120}{90} e^{-\alpha z} e^{-j\beta z} e^{-j 12.12^\circ}$$

$$\langle \hat{S} \rangle = \frac{1}{2} \frac{120^2}{90^2} e^{-2\alpha z} \cos(12.12^\circ) \hat{a}_z = 78.22 e^{-2\alpha z} \hat{a}_z \text{ W/m}^2$$

Problem 8.14 $f = 10 \text{ kHz}$ $\omega = 2\pi f = 6.283 \times 10^4 \text{ rad/s}$ $\sigma = 0.01 \text{ S/m}$ $\epsilon_r = 9$, $\mu_r = 4$

$$\frac{\sigma}{\omega \epsilon} = 2000 \text{ very good conductor. } \hat{\epsilon} = \epsilon [1 - j \frac{\sigma}{\omega \epsilon}] = 7.959 \times 10^{-11} - j 1.592 \times 10^{-7}$$

$$\hat{\gamma} = j\omega \sqrt{\mu \hat{\epsilon}} = 0.04 + j 0.04 \quad \alpha = 0.04 \text{ Np/m} \quad \beta = 0.04 \text{ rad/m} \quad \delta = \frac{1}{\alpha} = 25 \text{ m}$$

$$\hat{\eta} = \sqrt{\mu / \hat{\epsilon}} = 5.62 \angle 44.99^\circ \Omega, \quad u_p = \omega / \beta = 1.57 \times 10^6 \text{ m/s} \quad \lambda = \frac{2\pi}{\beta} = 157.08 \text{ m}$$

$$\tilde{E}_x = 100 e^{-\alpha z} e^{-j\beta z} \text{ V/m}, \quad \tilde{H}_y = \frac{100}{5.62} e^{-\alpha z} e^{-j\beta z} e^{-j 44.99^\circ} \text{ A/m}$$

$$\langle \hat{S} \rangle = \frac{1}{2} \times \frac{100^2}{5.62^2} e^{-2\alpha z} \cos(44.99^\circ) \hat{a}_z = 629.27 e^{-2\alpha z} \hat{a}_z \text{ W/m}^2$$

Problem 8.15 $\alpha = 77.485 \text{ Np/m}$ $\beta = 203.8 \text{ rad/m}$, $\hat{\gamma} = 77.485 + j 203.8 \text{ m}^{-1}$

$$\hat{\epsilon} = - \frac{(\alpha + j\beta)^2}{\omega^2 \mu_0} = (7.162 - j 6.366) \times 10^{-10}$$

$$\omega = 2\pi \times 10^9 \text{ rad/s}$$

$$\epsilon = 7.162 \times 10^{-10} \Rightarrow \epsilon_r = \epsilon / \epsilon_0 \approx 81.$$

$$\frac{\sigma}{\omega} = 6.366 \times 10^{-10} \Rightarrow \sigma = 4 \text{ S/m}$$

Propagation in y-direction:

$$\hat{\eta} = \sqrt{\frac{\mu_0}{\hat{\epsilon}}} = 36.213 \angle 20.82^\circ \Omega$$

$$\tilde{H}_x = 0.1 e^{-\alpha y} e^{-j\beta y} \text{ A/m}$$

$$\tilde{E}_z = 3.621 e^{-\alpha y} e^{-j\beta y} e^{j 20.82^\circ} \text{ V/m}$$

$$\left. \begin{array}{l} \tilde{H}_x = 0.1 e^{-\alpha y} e^{-j\beta y} \text{ A/m} \\ \tilde{E}_z = 3.621 e^{-\alpha y} e^{-j\beta y} e^{j 20.82^\circ} \text{ V/m} \end{array} \right\} \Rightarrow \langle \hat{S} \rangle = \frac{1}{2} \times 0.1 \times 3.621 e^{-2\alpha y} \cos(20.82^\circ) \hat{a}_y$$

$$= 0.169 e^{-154.97 y} \hat{a}_y \text{ W/m}^2$$

Problem 8.16

$$f = 10 \text{ kHz} \quad \omega = 2\pi f = 6.283 \times 10^4 \text{ rad/s} \quad \mu_r = 1$$

$$\epsilon_r = 1$$

$$\sigma = 5.8 \times 10^7 \text{ S/m} \quad \frac{\sigma}{\omega \epsilon} = 1.044 \times 10^{14} \quad \hat{\epsilon} = -j \frac{\sigma}{\omega} = -j 923.099$$

$$\hat{\gamma} = j\omega \sqrt{\mu_0 \hat{\epsilon}} = 1513 + j1513 \text{ m}^{-1} \quad \alpha = 1513 \text{ Np/m} \quad \beta = 1513 \text{ rad/m}$$

$$\hat{\eta} = \sqrt{\mu_0 / \hat{\epsilon}} = 3.64 \times 10^{-5} \angle 45^\circ \Omega \quad \delta = 1/\alpha = 6.609 \times 10^{-4} \text{ m} \approx 0.66 \text{ mm}$$

$$\vec{E}_x = 100 e^{-1513z} e^{-j1513z} \text{ V/m} \quad \vec{H}_y = 2.71 \times 10^6 e^{-1513z} e^{-j1513z} e^{-j45^\circ} \text{ A/m}$$

$$\vec{S} = \sigma \vec{E} \Rightarrow \vec{S}_x = 5.8 \times 10^9 e^{-1513z} e^{-j1513z} \text{ A/m}^2$$

$$\text{at } z = 0.2\delta, \quad E_x = 8187 \cos(\omega t - 11.46^\circ) \text{ V/m} \quad J_x = 4.75 \times 10^9 \cos(\omega t - 11.46^\circ) \frac{\text{A}}{\text{m}^2}$$

$$H_y = 2.219 \times 10^6 \cos(\omega t - 56.46^\circ) \text{ A/m}$$

Problem 8.17

$$\vec{H}_x = 0.1 e^{-15z} e^{-j15z} \text{ A/m}, \quad \alpha = \beta = 15 \Rightarrow \text{Very good conductor}$$

$$\omega = 2\pi \times 10^8 \text{ rad/s} \quad \sigma / \omega \epsilon \gg 1 \Rightarrow \hat{\epsilon} \approx \frac{\sigma}{j\omega} \quad \begin{matrix} \epsilon \rightarrow \epsilon_0 \\ \mu \rightarrow \mu_0 \end{matrix}$$

$$\hat{\gamma} = j\omega \sqrt{\mu_0 \hat{\epsilon}} = \sqrt{j\omega \mu_0 \sigma} \Rightarrow \alpha = \sqrt{\frac{\omega \mu_0 \sigma}{2}} \quad \text{or} \quad \sigma = \frac{2\alpha^2}{\omega \mu_0} = 0.57 \text{ S/m}, \quad \hat{\epsilon} = \frac{\sigma}{j\omega} = -j 9.071 \times 10^{-10}$$

$$\hat{\eta} = \sqrt{\mu_0 / \hat{\epsilon}} = 37.22 \angle 45^\circ \Rightarrow \vec{E}_y = -3.722 e^{-15z} e^{-j15z} e^{-j45^\circ} \text{ V/m}$$

$$\langle \hat{S} \rangle = \frac{1}{2} \times 3.722 \times 0.1 \cos 45^\circ e^{-30z} \hat{a}_z = 0.132 e^{-30z} \hat{a}_z \text{ W/m}^2$$

$$\text{at } z=0 \quad \langle \hat{S} \rangle_{z=0} = 0.132 \frac{\text{W}}{\text{m}^2}, \quad \text{at } z=\delta: \quad \langle \hat{S} \rangle \Big|_{z=\delta} = 0.018 \text{ W/m}^2$$

$$\text{or } z=1/\alpha$$

Hence, power lost per unit area: $0.132 - 0.018 = 0.114 \text{ W/m}^2$

Problem 8.18

$$\hat{\epsilon} = \epsilon [1 - j \tan \delta] \quad \text{where } \tan \delta = \frac{\sigma}{\omega \epsilon}$$

$$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu}{\epsilon}} \sqrt{\frac{1}{1 - j \tan \delta}}$$

$$1 - j \tan \delta = \sqrt{1 + \tan^2 \delta} \angle -\delta$$

$$= \sqrt{\frac{\mu}{\epsilon \sec \delta}} \angle -\delta/2$$

$$= \sec \delta \angle -\delta$$

$$\hat{\eta} = \eta \angle \theta_\eta, \quad \text{then} \quad \eta = \sqrt{\frac{\mu}{\epsilon \sec \delta}} \quad \text{and} \quad \theta_\eta = \frac{\delta}{2}$$

Problem 8.19 $f = 20 \text{ MHz}$, $\omega = 2\pi f = 40\pi \times 10^6 \text{ rad/s}$

When $z=0$ $|\vec{E}| = E_0$, When $z=1 \text{ m}$, $|\vec{E}| = 0.8 E_0 \Rightarrow e^{-\alpha(1)} = 0.8 \Rightarrow \alpha = 0.223 \text{ Np/m}$

$\hat{\eta} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu}{|\hat{\epsilon}|}} \angle 30^\circ \Rightarrow \hat{\epsilon} = |\hat{\epsilon}| \angle -40^\circ$

$\alpha = \text{Re}[\hat{\gamma}] = \text{Re}[j\omega\sqrt{\mu\hat{\epsilon}}] = \text{Re}[j\omega\sqrt{\mu}|\hat{\epsilon}| \angle -20^\circ]$
 $= \omega\sqrt{\mu}|\hat{\epsilon}| \cos 70^\circ \Rightarrow \sqrt{|\hat{\epsilon}|} = \frac{0.223}{\cos 70^\circ \omega\sqrt{\mu_0}} \Rightarrow |\hat{\epsilon}| = 2.1423 \times 10^{-11}$

$\hat{\epsilon} = 2.1423 \times 10^{-11} \angle -40^\circ$

$= (1.641 - j1.377) \times 10^{-11} \Rightarrow \epsilon_r = \frac{1.641 \times 10^{-11}}{\epsilon_0} = 1.856$

$\frac{\sigma}{\omega} = 1.377 \times 10^{-11} \Rightarrow \sigma = 1.73 \times 10^{-3} \text{ S/m}$

Problem 8.20 $\hat{\eta} = 60\pi \angle 30^\circ \Omega$ η_{mu} , $\hat{\eta} = \sqrt{\frac{\mu_0}{\hat{\epsilon}}} \Rightarrow \hat{\epsilon} = \frac{\mu_0}{\hat{\eta}^2} = [1.768 - j3.063] \times 10^{-11}$
 $\mu = \mu_0$ $\eta = 60\pi$

$\hat{\gamma} = j\omega\sqrt{\mu_0\hat{\epsilon}} = j\frac{\omega\mu_0}{\hat{\eta}} = \frac{\omega\mu_0}{\hat{\eta}} \cos 60^\circ + j \frac{\omega\mu_0}{\hat{\eta}} \sin 60^\circ$ $\epsilon_r = \frac{1.768 \times 10^{-11}}{\epsilon_0} = 2$

Since $\beta = 1.2 \text{ rad/s}$, $\omega = \frac{\beta\eta}{\mu_0 \sin 60^\circ} = 207.846 \times 10^6 \text{ rad/s}$ or $f = 33.08 \text{ MHz}$

$\alpha = \frac{\omega\mu_0}{\hat{\eta}} \cos 60^\circ = 0.693 \text{ Np/m}$

Attenuation in dB: $20 \log_{10} e^{0.693} = 20 \times 0.693 \log_{10} e = 20 \times 0.693 \times 0.4343 = 6.02 \text{ dB/m}$

Problem 8.21 $\mu_r = 1$ $\epsilon_r = 16$ $\sigma = 0.02 \text{ S/m}$ $f = 500 \text{ kHz}$
 $\omega = 3.142 \times 10^6 \text{ rad/s}$

$\hat{\epsilon} = \epsilon [1 - j \frac{\sigma}{\omega\epsilon}] = 1.415 \times 10^{-10} - j 6.366 \times 10^{-9}$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = 0.196 + j 0.201 \text{ m}^{-1}$ $\alpha = 0.196 \text{ Np/m}$, $\beta = 0.201 \text{ rad/m}$

$u_p = \frac{\omega}{\beta} = 1.524 \times 10^7 \text{ m/s}$

$\delta = \frac{1}{\alpha} = 5.09 \text{ m}$

$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = 14.048 \angle 44.36^\circ \Omega$

$\vec{E} = 120 e^{-0.196z} e^{-j0.201z} \hat{a}_x \text{ V/m}$

$\vec{H} = \frac{120}{14.048} e^{-0.196z} e^{-j0.201z} e^{-j44.36^\circ} \hat{a}_y \text{ A/m}$

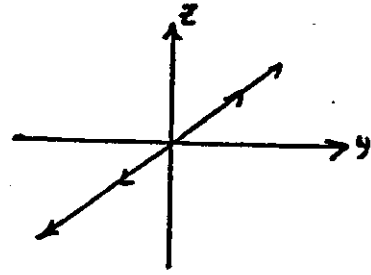
$\langle \hat{S} \rangle = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = 366.4 e^{-0.392z} \hat{a}_z \text{ W/m}^2$

At $z=d$, only 10% is left $\Rightarrow e^{-0.196d} = 0.1$ or $d = 11.75 \text{ m}$

Problem 8.22 a) At $x=0$

$$\left. \begin{aligned} E_y &= 100 \cos \omega t \\ E_z &= 100 \cos \omega t \end{aligned} \right\} \Rightarrow E_y = E_z$$

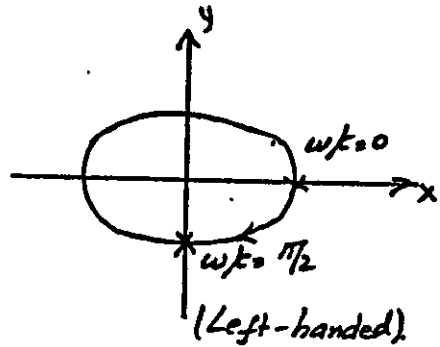
Linear polarization



b. When $\omega z = \pi/4$

$$\left. \begin{aligned} E_x &= 16 \cos \omega t \\ E_y &= -9 \sin \omega t \end{aligned} \right\} \Rightarrow \left(\frac{E_x}{16} \right)^2 + \left(\frac{E_y}{9} \right)^2 = 1$$

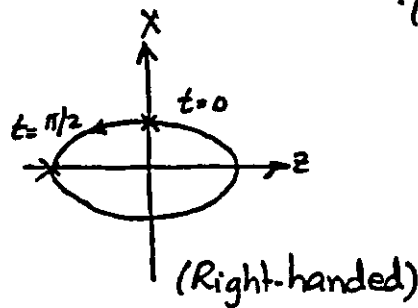
(Elliptical)



c) In a $y=0$ plane:

$$\left. \begin{aligned} E_x &= 3 \cos t \\ E_z &= -4 \sin t \end{aligned} \right\} \Rightarrow \left(\frac{E_x}{3} \right)^2 + \left(\frac{E_z}{4} \right)^2 = 1$$

Elliptical



Problem 8.23

$$\omega = 2 \times 10^8 \text{ rad/s}$$

$$\epsilon_r = 2.5$$

$$\lambda = \frac{2\pi}{\beta} = 5.96 \text{ m}$$

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_r} = 1.054 \text{ rad/m}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = 238.43 \Omega$$

Use Superposition Theorem:

$$\tilde{E}_x = 12 e^{-j\beta z} \Rightarrow \tilde{H}_y = \frac{12}{\eta} e^{-j\beta z}$$

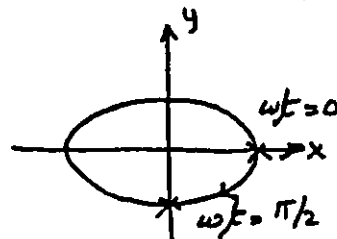
$$\tilde{E}_y = j5 e^{-j\beta z} \Rightarrow \tilde{H}_x = -j \frac{5}{\eta} e^{-j\beta z}$$

$$\vec{H} = \frac{1}{\eta} [12 e^{-j\beta z} \vec{a}_y - j5 e^{-j\beta z} \vec{a}_x]$$

$$\vec{H} = 0.05 \cos(\omega t - \beta z) \vec{a}_y + 0.02 \sin(\omega t - \beta z) \vec{a}_x \quad \text{A/m}$$

$$\text{at } z=0 \quad E_x = 12 \cos \omega t, \quad E_y = -5 \sin \omega t$$

$$\left(\frac{E_x}{12} \right)^2 + \left(\frac{E_y}{5} \right)^2 = 1 \quad \text{Elliptical.}$$



Left-handed

Problem 8.24

$$\vec{E} = E_0 e^{-j\beta z} \vec{a}_x + E_0 e^{-j\beta z} \vec{a}_y$$

$$= (a e^{-j\beta z} \vec{a}_x + j a e^{-j\beta z} \vec{a}_y) + (b e^{-j\beta z} \vec{a}_x - j b e^{-j\beta z} \vec{a}_y)$$

Then: $a + b = E_0$ and $a - b = -j E_0 \Rightarrow a = \frac{E_0}{\sqrt{2}} \angle -45^\circ$ $b = \frac{E_0}{\sqrt{2}} \angle 45^\circ$

Thus: $\vec{E} = \frac{E_0}{\sqrt{2}} [(\cos(\omega t - \beta z - 45^\circ) \vec{a}_x + \cos(\omega t - \beta z + 45^\circ) \vec{a}_y)$
 $+ (\cos(\omega t - \beta z + 45^\circ) \vec{a}_x + \cos(\omega t - \beta z - 45^\circ) \vec{a}_y)]$

In a $\beta z = 45^\circ$ plane:

$$\vec{E} = \frac{E_0}{\sqrt{2}} [\sin \omega t \vec{a}_x + \cos \omega t \vec{a}_y] + \frac{E_0}{\sqrt{2}} [\cos \omega t \vec{a}_x + \sin \omega t \vec{a}_y]$$

Right-handed Circularly polarized wave \swarrow Left-handed Circularly polarized

Problem 8.25

$$\vec{E} = 3E_0 e^{-j\beta z} \vec{a}_x - j4E_0 e^{-j\beta z} \vec{a}_y$$

$$= a e^{-j\beta z} \vec{a}_x - j a e^{-j\beta z} \vec{a}_y + b e^{-j\beta z} \vec{a}_x + j b e^{-j\beta z} \vec{a}_y$$

Thus $a + b = 3E_0$ and $a - b = 4E_0 \Rightarrow a = 3.5E_0$ and $b = -0.5E_0$

Time Domain:

$$\vec{E} = 3.5E_0 [\cos(\omega t - \beta z) \vec{a}_x + \sin(\omega t - \beta z) \vec{a}_y]$$

$$+ 0.5E_0 [-\cos(\omega t - \beta z) \vec{a}_x + \sin(\omega t - \beta z) \vec{a}_y]$$

In a $\beta z = 0$ plane:

$$\vec{E} = 3.5E_0 \cos \omega t \vec{a}_x + 3.5E_0 \sin \omega t \vec{a}_y \leftarrow \text{Right-handed Circularly polarized wave}$$

$$+ 0.5E_0 \cos \omega t \vec{a}_x + 0.5E_0 \sin \omega t \vec{a}_y \leftarrow \text{Left-handed Circularly polarized wave}$$

The two waves have different amplitudes.

Problem 8.26

$$f = 100 \text{ MHz} \quad \omega = 2\pi f = 6.283 \times 10^8 \text{ rad/s}$$

$$\epsilon_{r1} = 2.25 \quad \mu_{r1} = 1 \quad \sigma_1 = 2 \text{ mS/m} \quad \tan \delta_1 = \frac{\sigma_1}{\omega \epsilon_1} = 0.16$$

$$\epsilon_{r2} = 1 \quad \mu_{r2} = 1 \quad \sigma_2 = 20 \text{ mS/m} \quad \tan \delta_2 = \frac{\sigma_2}{\omega \epsilon_2} = 3.6$$

$$\hat{\gamma}_1 = j \frac{\omega}{c} \sqrt{\mu_{r1} \epsilon_{r1}} \sqrt{1 - j \tan \delta_1} = 0.251 + j3.152 \text{ m}^{-1} \quad \alpha_1 = 0.251 \text{ NP/m}, \beta_1 = 3.152 \text{ rad/m}$$

$$\hat{\gamma}_2 = j \frac{\omega}{c} \sqrt{\mu_{r2} \epsilon_{r2}} \sqrt{1 - j \tan \delta_2} = 2.45 + j3.223 \text{ m}^{-1}, \alpha_2 = 2.45 \text{ NP/m}, \beta_2 = 3.223 \text{ rad/m}$$

$$\hat{\eta}_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = 120\pi \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}} \sqrt{\frac{1}{1 - j \tan \delta_1}} = 249.744 \angle 4.55^\circ \Omega, \quad \hat{\eta}_2 = 120\pi \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}} \sqrt{\frac{1}{1 - j \tan \delta_2}}$$

$$\hat{P} = \frac{\hat{\eta}_2 - \hat{\eta}_1}{\hat{\eta}_2 + \hat{\eta}_1} = 0.318 \angle 114.8^\circ, \quad \hat{T} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} = 0.913 \angle 18.41^\circ = 195.634 \angle 37.24^\circ \Omega$$

$$\vec{E}_i = 10 e^{-0.251z} e^{-j3.152z} \vec{a}_x \text{ V/m}, \quad \vec{H}_i = \frac{10}{249.744} e^{-0.251z} e^{-j3.152z} e^{-j4.55^\circ} \vec{a}_y \text{ A/m}$$

$$\vec{E}_r = 3.178 e^{0.251z} e^{j3.152z} e^{j114.8^\circ} \vec{a}_x \text{ V/m}, \quad \vec{H}_r = -\frac{3.178}{249.744} e^{0.251z} e^{j3.152z} e^{j114.8^\circ} e^{-j4.55^\circ} \vec{a}_y \text{ A/m}$$

$$\vec{E}_t = 9.133 e^{-2.45z} e^{-j3.223z} e^{j18.41^\circ} \vec{a}_x \text{ V/m}$$

$$\vec{H}_t = \frac{9.133}{195.634} e^{-2.45z} e^{-j3.223z} e^{j18.41^\circ} e^{-j37.24^\circ} \vec{a}_y \text{ A/m} \quad \left. \begin{array}{l} \vec{E}_t \\ \vec{H}_t \end{array} \right\} \rightarrow \langle \hat{S}_t \rangle = \frac{1}{2} \text{Re} [\vec{E}_t \times \vec{H}_t^*] = 0.17 e^{-4.9z} \vec{a}_z \text{ W/m}^2$$

$$\langle \hat{S}_i \rangle = \frac{1}{2} \text{Re} [\vec{E}_i \times \vec{H}_i^*] = 0.2 e^{-0.502z} \vec{a}_z \text{ W/m}^2, \quad \langle \hat{S}_r \rangle = -0.02 e^{0.502z} \vec{a}_z \text{ W/m}^2$$

Problem 8.27

$$f = 200 \text{ MHz} \quad \omega = 2\pi f = 1.257 \times 10^9 \text{ rad/s}$$

$$\epsilon_{r1} = 1 \quad \mu_{r1} = 1 \quad \sigma_1 = 0.04 \text{ S/m}, \quad \epsilon_{r2} = 1, \quad \mu_{r2} = 1 \quad \sigma_2 = 4 \text{ S/m}$$

$$\hat{\epsilon}_1 = \epsilon_0 [1 - j \frac{\sigma_1}{\omega \epsilon_0}] = 8.842 \times 10^{-12} - j3.183 \times 10^{-11}, \quad \hat{\epsilon}_2 = \epsilon_0 [1 - j \frac{\sigma_2}{\omega \epsilon_0}] = 8.842 \times 10^{-12} - j3.183 \times 10^{-9}$$

$$\hat{\gamma}_1 = j\omega \sqrt{\mu_0 \hat{\epsilon}_1} = 4.9 + j6.446 \text{ m}^{-1}$$

$$\hat{\gamma}_2 = j\omega \sqrt{\mu_0 \hat{\epsilon}_2} = 56.121 + j56.277 \text{ m}^{-1}$$

$$\hat{\eta}_1 = \sqrt{\mu_0 / \hat{\epsilon}_1} = 195.634 \angle 37.24^\circ \Omega$$

$$\hat{\eta}_2 = \sqrt{\mu_0 / \hat{\epsilon}_2} = 19.869 \angle 44.92^\circ \Omega$$

$$\hat{P} = \frac{\hat{\eta}_2 - \hat{\eta}_1}{\hat{\eta}_2 + \hat{\eta}_1} = 0.817 \angle 178.43^\circ$$

$$\hat{T} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} = 0.185 \angle 6.97^\circ$$

$$\vec{E}_i = 50 e^{-4.9z} e^{-j6.446z} \vec{a}_x \text{ V/m}$$

$$\vec{H}_i = \frac{50}{195.034} e^{-4.9z} e^{-j6.446z} e^{-j37.24^\circ} \vec{a}_y \text{ A/m} \quad \langle \hat{S}_i \rangle = \frac{1}{2} \text{Re}[\vec{E}_i \times \vec{H}_i^*] = 5.103 e^{-9.8z} \vec{a}_z \text{ W/m}^2$$

$$\vec{E}_r = 40.831 e^{4.9z} e^{j6.446z} e^{j178.42^\circ} \vec{a}_x \text{ V/m}$$

$$\vec{H}_r = -\frac{40.831}{195.034} e^{4.9z} e^{j6.446z} e^{j178.42^\circ} e^{-j37.24^\circ} \vec{a}_y \text{ A/m} \quad \langle \hat{S}_r \rangle = \frac{1}{2} \text{Re}[\vec{E}_r \times \vec{H}_r^*] = -3.403 e^{9.8z} \vec{a}_z \text{ W/m}^2$$

$$\vec{E}_t = 9.253 e^{-5.121z} e^{-j52.277z} e^{j6.97^\circ} \vec{a}_x \text{ V/m}$$

$$\vec{H}_t = \frac{9.253}{19.869} e^{-5.121z} e^{-j52.277z} e^{j6.97^\circ} e^{-j44.93^\circ} \vec{a}_y \text{ A/m}, \quad \langle \hat{S}_t \rangle = \frac{1}{2} \text{Re}[\vec{E}_t \times \vec{H}_t^*] = 1.525 e^{-10.242z} \vec{a}_z \text{ W/m}^2$$

$$\delta_2 = \frac{1}{\alpha_2} = 17.82 \text{ mm.}$$

Problem 8.28 $\omega = 90 \times 10^6 \text{ rad/s}$ $\sigma_2 = 0.4 \text{ S/m}$ $\mu_2 = \mu_0$ $\epsilon_2 = \epsilon_0$

$$\eta_1 = 377 \Omega \quad \beta_1 = \frac{\omega}{c} = 0.3 \text{ rad/m} \quad \tan \delta_2 = \frac{\sigma_2}{\omega \epsilon_2} = 6.206$$

$$\hat{\gamma}_2 = j \frac{\omega}{c} \sqrt{\mu_0 \epsilon_0} \sqrt{1 - j \tan \delta_2} = 4.389 + j5.153 \text{ m}^{-1} \quad \hat{\eta}_2 = 16708 \angle 40.42^\circ \Omega$$

$$\hat{\rho} = \frac{\hat{\eta}_2 - \eta_1}{\hat{\eta}_2 + \eta_1} = 0.935 \angle 176.7^\circ, \quad \hat{\tau} = \frac{2\hat{\eta}_2}{\eta_1 + \hat{\eta}_2} = 0.086 \angle 38.83^\circ$$

$$\vec{E}_i = [-j100 \vec{a}_x + 200 \vec{a}_y] e^{-j0.3z} \text{ V/m} \quad \langle \hat{S}_i \rangle = \frac{1}{2} \text{Re}[\vec{E}_i \times \vec{H}_i^*]$$

$$\vec{H}_i = \frac{-200 \vec{a}_x - j100 \vec{a}_y}{377} e^{-j0.3z} \text{ A/m} = 66.31 \vec{a}_z \text{ W/m}^2$$

$$\vec{E}_r = [-j93.5 \vec{a}_x + 187 \vec{a}_y] e^{j0.3z} e^{j176.7^\circ} \text{ V/m.} \quad \langle \hat{S}_r \rangle = \frac{1}{2} \text{Re}[\vec{E}_r \times \vec{H}_r^*]$$

$$\vec{H}_r = \frac{(187 \vec{a}_x + j93.5 \vec{a}_y)}{377} e^{j0.3z} e^{j176.7^\circ} \text{ A/m} = -57.97 \vec{a}_z \text{ W/m}^2$$

$$\vec{E}_t = (-j8.6 \vec{a}_x + 17.2 \vec{a}_y) e^{-4.389z} e^{-j5.153z} e^{j38.83^\circ} \text{ V/m}$$

$$\vec{H}_t = \frac{-17.2 \vec{a}_x + j8.6 \vec{a}_y}{16708} e^{-4.389z} e^{-j5.153z} e^{j38.83^\circ} e^{-j40.42^\circ} \text{ A/m}$$

$$\langle \hat{S}_t \rangle = \frac{1}{2} \text{Re}[\vec{E}_t \times \vec{H}_t^*] = 8.4 e^{-8.782z} \vec{a}_z \text{ W/m}^2$$

At the interface $z=0$: $\langle \hat{S}_i \rangle + \langle \hat{S}_r \rangle = 8.34 \vec{a}_z \text{ W/m}^2$
 $= \langle \hat{S}_t \rangle$

Problem 8.29 $\omega = 60 \times 10^6 \text{ rad/s}$ $\mu_{r1} = 1, \mu_{r2} = 1, \epsilon_{r1} = 9, \epsilon_{r2} = 1$

$\beta_1 = \frac{\omega}{c} \sqrt{\epsilon_{r1}} = 0.6 \text{ rad/m}$ $\beta_2 = \frac{\omega}{c} = 0.2 \text{ rad/m}$ $c = 3 \times 10^8 \text{ m/s}$

$\eta_1 = \frac{120\pi}{\sqrt{\epsilon_{r1}}} = 125.664 \Omega$ $\eta_2 = 120\pi \approx 377 \Omega$

$\rho = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = 0.5$ $\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 1.5$

$\vec{E}_i = j150 e^{-j0.6z} \vec{a}_x \text{ V/m}$ $\langle \hat{S}_i \rangle = \frac{1}{2} \text{Re}[\vec{E}_i \times \vec{H}_i^*] = 89.5 \vec{a}_z \text{ W/m}^2$
 $\vec{H}_i = j \frac{150}{\eta_1} e^{-j0.6z} \vec{a}_y \text{ A/m}$

$\vec{E}_r = -j75 e^{j0.6z} \vec{a}_x \text{ V/m}$

$\vec{H}_r = j \frac{75}{\eta_1} e^{j0.6z} \vec{a}_y \text{ A/m}$

$\langle \hat{S}_r \rangle = -22.4 \vec{a}_z \text{ W/m}^2$

$\vec{E}_t = -j225 e^{-j0.2z} \vec{a}_x \text{ V/m}$

$\vec{H}_t = -j \frac{225}{377} e^{-j0.2z} \vec{a}_y \text{ A/m}$

$\langle \hat{S}_t \rangle = 67.1 \vec{a}_z \text{ W/m}^2$

Problem 8.30 $\omega = 120 \times 10^6 \text{ rad/s}$ $c = 3 \times 10^8 \text{ m/s}$, $\epsilon_{r1} = 9, \epsilon_{r2} = 1, \mu_{r1} = \mu_{r2} = 1$

$\beta_1 = \frac{\omega}{c} \sqrt{\epsilon_{r1}} = 1.2 \text{ rad/m}$, $\eta_1 = \frac{120\pi}{\sqrt{\epsilon_{r1}}} = 125.664 \Omega$

$\beta_2 = \frac{\omega}{c} = 0.4 \text{ rad/m}$, $\eta_2 = 120\pi \approx 377 \Omega$

$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.5$ $\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = 1.5$

$\vec{E}_i = j50 e^{j1.2z} \vec{a}_x \text{ V/m}$, $\vec{H}_i = j \frac{50}{\eta_1} e^{j1.2z} \vec{a}_y \text{ A/m}$

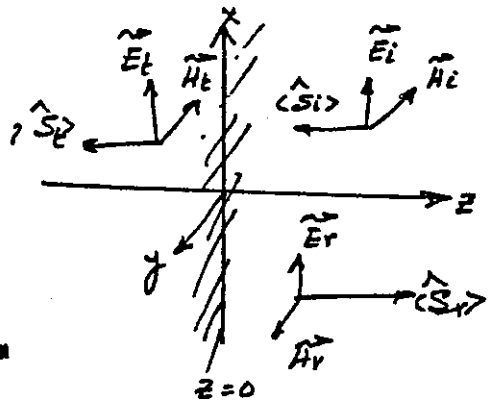
$\vec{E}_r = j25 e^{-j1.2z} \vec{a}_x \text{ V/m}$, $\vec{H}_r = j \frac{25}{\eta_1} e^{-j1.2z} \vec{a}_y \text{ A/m}$

$\vec{E}_t = j75 e^{j4z} \vec{a}_x \text{ V/m}$, $\vec{H}_t = j \frac{75}{377} e^{j4z} \vec{a}_y \text{ A/m}$

$\langle \hat{S}_i \rangle = \frac{1}{2} \text{Re}[\vec{E}_i \times \vec{H}_i^*] = -9.95 \vec{a}_z \text{ W/m}^2$

$\langle \hat{S}_r \rangle = \frac{1}{2} \text{Re}[\vec{E}_r \times \vec{H}_r^*] = 2.49 \vec{a}_z \text{ W/m}^2$

$\langle \hat{S}_t \rangle = \frac{1}{2} \text{Re}[\vec{E}_t \times \vec{H}_t^*] = -7.46 \vec{a}_z \text{ W/m}^2$



Problem 8.31 $P = 0.25 \Rightarrow \tau = 1.25$

Thus, $\frac{2\eta_2}{\eta_1 + \eta_2} = 1.25$ or $0.75\eta_2 = 1.25\eta_1 \Rightarrow \frac{\eta_2}{\eta_1} = 1.667 = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \Rightarrow \frac{\epsilon_1}{\epsilon_2} = 2.78$

$P = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = 0.25 \Rightarrow \frac{\eta_2/\eta_1 - 1}{\eta_2/\eta_1 + 1} > 0$ ($\because P > 0$). Thus, $\frac{\epsilon_1}{\epsilon_2} > 1$ i.e. $\epsilon_1 > \epsilon_2$

If $\epsilon_2 = 1.25\epsilon_0$, then $\epsilon_1 = 2.78\epsilon_0 = 3.47\epsilon_0$ Hence: $\epsilon_{r2} = 3.47$

Problem 8.32 $P = -0.75 \quad \tau = 0.25 = \frac{2\eta_2}{\eta_1 + \eta_2} \Rightarrow \frac{\eta_2}{\eta_1} = 0.143 = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$

Hence $\frac{\epsilon_1}{\epsilon_2} = 0.02$

Since $P < 0$, $\frac{\eta_2}{\eta_1} < 1$ or $\sqrt{\frac{\epsilon_1}{\epsilon_2}} < 1 \Rightarrow \epsilon_2 > \epsilon_1$

Let $\epsilon_1 = 2.25\epsilon_0$, then $\epsilon_2 = 112.5\epsilon_0$.

Problem 8.33 $c = 3 \times 10^8 \text{ m/s}$

$\beta_1 \cos \theta_1 = 0.766 \quad \beta_1 \sin \theta_1 = 0.643 \Rightarrow$

$\beta_1 = 1 \text{ rad/m}$ and $\theta_1 = 40^\circ$

$\omega = \beta_1 v_p = c\beta_1 = 3 \times 10^8 \text{ rad/s}$

$\beta_2 = \frac{\omega}{c} \sqrt{\epsilon_{r2}} = 1.5 \text{ rad/m}$

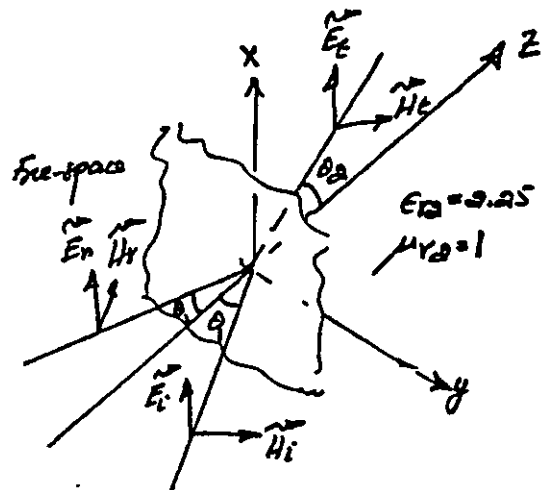
$\beta_1 \sin \theta_1 = \beta_2 \sin \theta_2 \Rightarrow \theta_2 = 25.38^\circ$

$\sin \theta_2 = 0.429 \quad \cos \theta_2 = 0.903$

$\eta_1 = 377\Omega \quad \eta_2 = \frac{120\pi}{\sqrt{2.25}} = 251.327\Omega$

$P_\eta = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} = -0.278$

$\tau = \frac{2\eta_2 \cos \theta_1}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2} = 0.722$



$\vec{E}_i = 50 e^{-j0.766z} e^{j0.643y} \vec{a}_x \text{ V/m}$

From $\nabla \times \vec{E} = -j\omega\mu_0 \vec{H}$,

$\vec{H}_i = (0.102 \vec{a}_y + 0.085 \vec{a}_z) e^{-j0.766z} e^{j0.643y} \text{ A/m}$

$\langle \vec{S}_i \rangle = \frac{1}{2} \text{Re} [\vec{E}_i \times \vec{H}_i^*]$

$= 2.55 \vec{a}_z - 2.13 \vec{a}_y \text{ W/m}^2$

$$\vec{E}_i = -13.9 e^{j0.766z} e^{j0.643y} \vec{a}_x \text{ V/m}$$

$$\langle \vec{S}_i \rangle = -0.19 \vec{a}_z - 0.17 \vec{a}_y \text{ W/m}^2$$

$$\vec{H}_i = (0.028 \vec{a}_y - 0.024 \vec{a}_z) e^{j0.766z} e^{j0.643y} \text{ A/m}$$

$$\vec{E}_t = 36.1 e^{-j1.355z} e^{j0.643y} \vec{a}_x \text{ V/m}$$

$$\langle \vec{S}_t \rangle = 2.35 \vec{a}_z - 1.12 \vec{a}_y \text{ W/m}^2$$

$$\vec{H}_t = (0.13 \vec{a}_y + 0.062 \vec{a}_z) e^{-j1.355z} e^{j0.643y} \text{ A/m}$$

Problem 8.34

$$\eta_1 = 377 \Omega \quad \eta_2 = 0 \quad \beta_1 = 1 \text{ rad/m} \quad \omega = 3 \times 10^8 \text{ rad/s} \quad P_n = -1$$

$$\vec{E}_i = 50 e^{-j0.766z} e^{j0.643y} \vec{a}_x \text{ V/m}$$

$$\vec{E}_r = -50 e^{j0.766z} e^{j0.643y} \vec{a}_x \text{ V/m}$$

$$\vec{E} = \vec{E}_i + \vec{E}_r = -j100 \sin(0.766z) e^{j0.643y} \vec{a}_x \text{ V/m}$$

$$\vec{H}_i = (0.102 \vec{a}_y + 0.085 \vec{a}_z) e^{-j0.766z} e^{j0.643y} \text{ A/m}$$

$$\vec{H}_r = (0.102 \vec{a}_y - 0.085 \vec{a}_z) e^{j0.766z} e^{j0.643y} \text{ A/m}$$

$$\vec{H} = \vec{H}_i + \vec{H}_r = 0.204 \cos(0.766z) e^{j0.643y} \vec{a}_y - j0.17 \sin(0.766z) e^{j0.643y} \vec{a}_z \text{ A/m}$$

$$\text{at } z=0 \quad \vec{H}(0) = 0.204 e^{j0.643y} \vec{a}_y$$

$$\vec{a}_n \times \vec{H}(0) = \vec{J}_s(0) \Rightarrow$$

$$\vec{J}_s(0) = 0.204 e^{j0.643y} \vec{a}_n$$

